Coordination modes in the multisegmental dynamics of hula hooping

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Abstract. In hula hooping, organized motions of the body keep the hoop in stable oscillatory motion parallel to the ground. We examined the hypothesis that the multiple degrees of freedom (DF) of the lower limbs in producing the oscillations are resolved into a few control DF. The Karhunen-Loève decomposition was applied to the kinematics of the lower limbs in three experiments in which oscillation amplitude and frequency were manipulated. Kinematic variance was accommodated by two modes whose relative contributions varied with task parameters. Complementary analyses of interjoint Hilbert relative phase suggested a lower-limb organization into a vertical suspension mode and an oscillatory fore-aft mode. These modes might stabilize the hoop's angular momentum by controlling, respectively, its vertical and horizontal components.

1 Introduction

The systematic coordination of multiple body segments characterizes everyday activities such as walking, swimming, reaching, and manipulating. Frequently, these multiple segments are constrained to perform tasks that involve balancing or controlling unstable objects as in riding a bicycle, for example, or even writing with a pencil. In such cases, the equations of motion of the object controlled through spatial and temporal changes in segmental organization are either unknown or not well understood.

"Hula hooping" is a complex skill in which an unstable object, a hoop, is kept in steady oscillation parallel with the ground plane by means of coordinate oscillations of the body. The physical basis of the skill is

the conservation of angular momentum. In manipulating the hoop, the performer exerts small but carefully regulated impulses (where impulse equals force × time) by allowing the body to impinge on a small portion of the interior periphery of a short section of the hoop. The subtle application of impulses produces changes in the angular momentum of the hoop. If the impulse forces are so directed that there is a small vertical component of this momentum opposing the force of gravity (which acts uniformly over the plane of the hoop), then the resulting horizontal motion will be maintained.

Generalizations of the skill of hula hooping entail variations in the size of the hoop, the frequency of hoop oscillations, the segment of the body about which the hoop oscillates (e.g., neck, chest, waist), and the number of hoops oscillated simultaneously. In the skill's most common form, hoop oscillations occur about the waist and involve concurrent oscillatory motions of the hips, knees, and ankles. The achievement of a particular pattern of sustained, coupled oscillations about these joints is key in the maintenance of the hoop in dynamic equilibrium. A systematic analysis of the act of balancing a hoop at the waist through movements of the lower limbs may be an ideal candidate, therefore, for understanding the strategies used by the central nervous system in (a) controlling an unstable object and (b) constraining multiple degrees of freedom in order to sustain a particular pattern of oscillations about several joints.

The starting point for the present research is the hypothesis that the control basis for a skilled behavior is of reduced dimensionality relative to the number of dimensions needed to express the behavior's kinematics. The goal of the research was to resolve the number of lower-limb coordination modes required for successful hula hooping under variations in hoop size and oscillation frequency. The research can be viewed as spadework for identifying the task's functional description thus adding to our knowledge of the nature of control structures for coordination and how the variability of these structures or manifolds might be actively or

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optimally used by the CNS in organizing movements (Danion et al. 2003; Scholz and Schöner 1999; Todorov and Jordan 2002).

The relation between neural activity, muscular activity, and movement is equivocal (Berkinblit et al. 1986; Bernstein 1967; Feldman and Levin 1995; Turvey 1990). On the basis of the constantly changing interactions among neural and muscular components, a given pattern of muscular activity might give rise to different movements, and different patterns of muscular activity might give rise to the same movement. Likewise, different movements of the body can have the same kinetic consequences. For example, in balancing a hoop, a particular relation among the patterns of neuromuscular activities at the ankles, knees, and hips need not, on each occasion, produce the same hoop motion and vice versa.

The formation of movement trajectories by a system of multiple joints poses additional concerns (for a review, see Latash 1993). In particular, the relation between trajectories in joint coordinates and the position of the end effectors in extrinsic coordinates is ambiguous (Polit and Bizzi 1978; Lacquaniti and Soechting 1982; Feldman 1986; Balasubramaniam and Feldman 2003). In coordinating the lower limbs, for example, the total number of relations that can exist among the ankles, knees, and hips is very large, even in the presence of biomechanical restrictions. It is generally presumed that there is a principled system of constraints such that only the right movement outcomes are realized (Greene 1975).

Two dominant approaches to the special problems of multisegmental coordination, such as those posed by hula hooping, can be identified. The *neuromuscular approach* frames the coordination problem as the discovery of the appropriate structural variables controlled by the neurophysiological apparatus (Nashner and McCollum 1985; Latash 1993). The *dynamical systems approach* assumes that general principles of coordination emerge when the movements are treated as the solutions of a self-organizing dynamical system (Kugler and Turvey 1987; Turvey 1990; Kelso 1995; Haken 1996).

1.1 The neuromuscular approach and hypothesized control strategies

The control of upright posture is one of the most widely studied areas in multisegmented movement control and thus lends itself as a good starting point in the study of lower-limb dynamics (Balasubramaniam and Wing 2002). Arguably, the major theoretical position is that promoted by Nashner and colleagues (Nashner and McCollum 1985). Preserving upright stance in the face of perturbations is based in the organizational schemes of the neuromuscular system. The particular patterns of postural activity are determined by the relations among the CNS, the musculature it controls, and sensory information. These organizational schemes are manifest primarily in the relations among ankle, knee, and hip joints.

Nashner and McCollum (1985) identified two dominant strategies used to return the body to equilibrium when perturbed: the *ankle strategy* and the *hip strategy*.

The ankle strategy is usually seen when the support surface is firm and the perturbations are weak. The center of mass (CoM) of the body is adjusted by rotation of the body about the ankle joints, achieved by the relevant muscle groups contracting in a distal-to-proximal pattern. In the ankle mode, the motion of the segments of the body is presumed to act like an inverted pendulum with no functional movement around the hips. The hip strategy, or the predominant use of hip rotations to return to stability when perturbed, is typically observed when the perturbations are rapid and large and the support structure is compliant or smaller than the feet. Exclusion of a knee strategy is due to the fact that knee movements tend to affect the height of the CoM more than influencing lateral position, which is predominantly controlled by ankle and hip rotations.

The neuromuscular organization of modular or discrete "synergies" proposed by Nashner and colleagues suggests that postural synergies are organized spinally by modular function generators and are automatically triggered by features of somatosensory input that comes from proprioceptive information related to joint angular rotations. The modular synergy dedicated to anteroposterior (AP) sway, for example, is activated in proportion to ankle rotational input. In contrast, the vertical suspensory synergy module is activated in proportion to knee rotational input. Typically, the AP sway module inhibits the suspensory module so that the two synergies are not triggered simultaneously.

The neuromuscular approach suggests that the coordination observed at the level of the limbs is a consequence of more basic patterns of coordination defined over the musculature, which in turn depend on neural activation patterns. This approach also hypothesizes that the optimization and selection of action is based on principled minimization of muscle recruitment, precision and speed of neural calculations, etc. Thus an infinite number of possible patternings among limb segments get constrained to a few neuromuscular organization schemes or synergies.

The appeal of the neuromuscular approach comes from the fact that it succeeds to some degree in the reduction of the dimensionality of the movement system (Kay 1988; Turvey 1990). Hip and ankle strategies help compress a high-dimensional postural control space, with many DF at the muscular and joint level, into a lower dimensional space, with fewer interacting DF. If the neuromuscular synergy approach were extended to the task of balancing a hoop, one might propose that there should be simultaneous activation of ankle, knee, and hip rotations produced in proportion to their levels of disturbance from equilibrium. But such an approach would not be able to identify what kinds of continuous control mechanisms would have to be exerted in order to monitor a fixed relation between the joints under dynamically evolving conditions (Saltzman and Kelso 1987). The levels of constraint identified in the case of upright stance might not be relevant under conditions that demand continuous task-specific postural adjustments.

Recent work by Bardy et al. (1999) has challenged the basic conceptual structure of Nashner's neuromuscular

approach. Participants had to track visual targets with their head over variations in mechanical factors such as effective foot size and height of CoM. In cases where only ankle motion would be prescribed by Nashner's scheme, Bardy et al. found, in addition, substantial hip activity. Most significantly, they observed the emergence of two stable phase relations, 0 and π radians, between ankle and hip motions influenced by the imposed task constraints. Bardy et al. were led to the conclusion that postural coordination modes are self-organized patterns, with the relative phasing of the hip and ankle characterizing the patterns.

From Bardy et al.'s perspective, one might view the control structure for the task of balancing a hoop as time-evolving patterns of relations among the joint segments as opposed to activation of joint rotations in proportion to deviation from equilibrium. In other words, the control process is that of discovering and assembling stable coordination modes rather than activating and combining fixed strategies. The following section details this alternative theoretical approach.

1.2 The dynamical systems perspective

Patently, the coordination of biological movements involves a large number of multiply nested (neural, muscular, skeletal) components. When the number of states of each of these components is taken into account, there are, arguably, too many DF in any given movement to make executive control possible (Bernstein 1967, 1996). It has been proposed that, during the performance of any given act, the very many components are combined into functional units called *coordinative structures* or *synergies* (e.g., Turvey et al. 1988; Kugler et al. 1980; Kay 1988; Mitra et al. 1998). Contrary to the neuromuscular approach (Nashner and McCollum 1985), in the dynamics perspective, the term synergy refers to a temporarily assembled functional unit. It is expressed in dynamical terms rather than anatomical or biomechanical terms; it is a collective, task-specific organizational state achieved by the system.

For a person hula hooping, synergies (coordinative structures) are the collective variables that capture the evolution of the spatiotemporal pattern involved in performing the task. Time variations about the hips, knees, and ankles in three dimensions yield 18 equations of motion. At one level of description, one might view the performance as solving a problem involving 18 variables. Alternatively, one might view it as the control of one point (or a few points) in an 18-dimensional space (e.g., Stewart 1989). The time-varying characteristics or equations of motion of this "one point" may be representative of the behavior of the 18 variables taken collectively (Haken 1977, 1983, 1996; Turvey 1998). Such a variable, which captures the spatiotemporal characteristics of a system of moving parts, is a collective variable or order parameter (Haken 1977; Kelso 1995; see Appendix A). In theory, therefore, the task of balancing a hoop may be represented by a motion equation expressing the time evolutionary characteristics of a collective variable. The solutions or zero crossings of this motion equation would define the stability of the task, a measure that would be expected to vary systematically over variants of the task (Turvey 1998).

Let us return to the indeterminism of the end effector-to-joint space mapping in trajectory formation. It has been shown that this mapping is ill defined because the joint angles are not functions of end-effector positions (Klein and Huang 1983; Feldman 1986; Buchanan et al. 1997). Importantly, however, when the system is allowed to evolve in time, that is, become a dynamical system, these indeterminacies are at least partly resolved (Buchanan et al. 1997). Soechting et al. (1986) found that a constant phase relation is maintained between forearm and upper arm elevation angles when circles and ellipses are drawn, even though static mappings do not exist between the arm configuration and end point. Temporal constraints are clearly implicated in the dynamics that govern the formation and change of behavioral patterns in a range of coordination tasks (for summaries see Walter and Swinnen 1992; Kelso 1995; Schmidt and Turvey 1995; Amazeen et al. 1998).

Also to be noted, the parameters that constrain such systems, often called *control parameters*, are typically nonspecific to the system itself. Systematic manipulation of such a control parameter produces qualitative changes in the space-time characteristics of the system's behavior as expressed through its order parameter. Usually control parameters in the synergetic approach are associated with the experimental induction of phase transitions. It is important to note that the concept of control parameters is given a broader interpretation in this paper.

In adopting the dynamical systems approach, the present research attempts to identify these collective spatiotemporal variables whose time evolution can reveal the stability, or lack thereof, of the movement patterns that constitute hula hooping. The goal of this research is therefore to identify topological organizations of the lower limbs that assist in the hula hooping rather than for the entire system encompassing the actor and the hoop.

1.3 Spatiotemporal patterns and the inverse self-organization problem

The first order of business in the study of topological organizations of the lower limbs is the identification of dynamical constraints that are specific to the task of hula hooping. The strategy of constraint identification involves the specification of the well-defined conditions that constrain the actor's movements for establishing an invariant pattern as in cascade juggling (Beek and van Santvoord 1992; Beek and Turvey 1992) or bouncing a ball (Sternad et al. 2001). Although the aforementioned two skills are qualitatively rather different, they both involve the control of an object where the velocity at impact or release fully determines the ball's flight trajectory. The identified constraints serve as the basis for the actor's boundaries for executing the task. The

questions that arise in studying the dynamics of hula hooping are in the spirit of those described above. What kinds of constraints organize the spatiotemporal patterning seen in the motion of the multiple-limb segments? Is it possible to identify coordination modes from the spatiotemporal variability of the system? What might constitute a collective variable for the task of hula hooping? In what space might one expect to find such a set of collective variables? Further, can one observe stable and unstable states in the patterning of the individual limb segments that constitute successful performance of the task?

The problem of finding the right collective description for a system is similar to a fairly ubiquitous problem in self-organizing connectionist networks (Ghahramani and Hinton 1998). A group of nodes in a layer of a neural network do not carry an explicit representation of the goal to be carried out. They are only aware of the local rules, if any exist, and the strength of their mutual relationships of cooperation and competition with other nodes in the network. The pattern of activation or distribution of the ever-changing weights is the only determinant of the states that the system is likely to settle into. The only way to identify if there was a spatiotemporal distribution in the activities of the nodes themselves is to see the system solve the problem that it is engaged in and then, from the node firing pattern, identify a possibly collective description.

Identifying a collective description for hula hooping is a problem of like kind. The individual oscillations about the ankles, knees, and hips are assembled only in relation to one another. It is the mutual understanding of the goal that constrains the multiple components of the hula hooping system. We watch the system perform and solve a problem successfully, and from the patterning of the oscillators with respect to each other we can identify constraints that reduce the number of interactions in the state space and find invariant patterns in the execution of the movements. This has been referred to as the *inverse self-organization problem* (Friedrich et al. 1998). A collective description of the system is made possible because of the cooperative behavior of the various components of the system that brings about a corresponding reduction in the dynamical DF of the system as constrained by a mutual understanding of the goal.

1.4 Determining the modes

In the present research, we used the Karhunen-Loève (K-L) transformation (otherwise referred to as *singular value decomposition* or *principal component analysis*) to reveal the modal organization of the lower limbs in hula hooping (see below for details). Methods like the K-L transformation have been used successfully for the purpose of dimensionality reduction in neural pattern formation (Bell and Sejnowksi 1995; Ghahramani and Hinton 1998), pattern recognition (Fukunaga 1990), three-ball cascade juggling (Post et al. 2000; Huys et al. 2002), performance on the pedalo (Haken 1996), playground swinging (Post et al. 2003), and analysis of

spatiotemporal patterns in speech production disorders (Herzel et al. 1994). A good review of the abovementioned technique may be found in Daffertshofer et al. (2003). With respect to the time series of six joints in three-dimensional motion shown in Fig. 1, the K-L transformation reduces the 18-component vector to a two- or three-component vector.

Of special interest to the present research was the question of whether the reduced description expected from the K-L transformation is sensitive to experimental manipulations of candidate control variables. In collective dynamics of interlimb coordination, the relative phase (difference between the phase angles of two limbs) has shown two modal states: in-phase (stability at $0°$) and antiphase (stability at $180°$). The persistence and degree of stability of these modal states depend on movement speed and the symmetry of the contributions of the two limbs to the coordination (Kelso 1995; Amazeen et al. 1998; Turvey 1998). The task of the present research was to see if the time evolution of the collective variables indexed by K-L modes of hula hooping exhibit changes in stability with variations in task demands that affect the segmental movement amplitudes and frequency. Of additional interest to the present research was the behavior of the phase relations that exist between and among the various moving segments. Do they undergo observable changes with changes in task parameters such as amplitude and frequency?

2 Experiment 1: How many modes?

Experiment 1 was directed at the expectation that a small number of orthogonal principal components are sufficient to describe the entire spatiotemporal patterning of the ankle, knee, and hip joints in hula hooping. It was also directed at the expectation, previewed above, that a large ensemble of time series of body segments, as in Fig. 1, could be fully expressed by a few modes. The expectation is well motivated by prior research. The
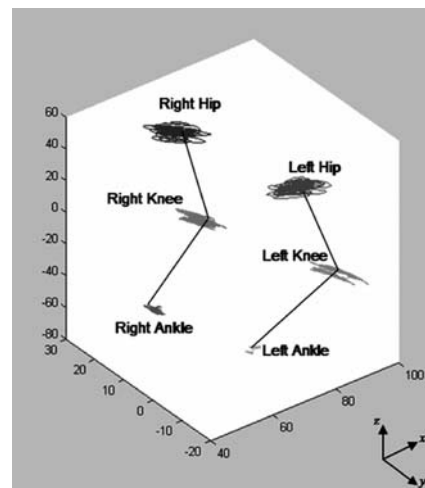
Fig. 1. Three-dimensional plot of the displacements in *x*, *y*, and *z* directions for a participant engaged in hula hooping for 20 s

complex skill of riding a pedalo (two steps fixed eccentrically to two wheels) is a whole-body task that requires, at least, a 22-dimensional vector to describe its kinematics. Application of the K-L transformation revealed that the skill was dominated by five or fewer modes in beginners and, perhaps, by simply one mode in the highly practiced participant (Haken 1996). Similarly, it has been shown that four K-L modes capture the spatiotemporal pattern of three-ball cascade juggling (Post et al. 2000).

2.1 Method

2.1.1 Participants.
Seven paid participants (four male and three female) served as subjects in this experiment. All participants were intermediate-level hula hoopers (none had performed competitively). None of the participants reported any recent injuries or disabilities. Their ages ranged from 12 to 27 years, with a mean of 18.4 years, and their heights ranged from 145 to 180 cm (mean of 164 cm) and weights from 38 to 65 kg (mean of 49.5 kg). The participants were chosen on the basis of a demonstrated ability to sustain hoop oscillations without losing control for a period of 30 s. The same participants returned for experiments 2 and 3.

2.1.2 Apparatus, data collection, and reduction.
The "hula hoop" (Wham-O Corporation, Torrance, CA) was 80.4 cm in diameter and weighed approximately 200 g.

Movements were recorded using a magnetic motion tracking system developed by Polhemus FASTRACK, Polhemus Corporation, Colchester, VT. 6-D Research system software was used to capture the data on a Pentium II-class microcomputer. Six tracking system receivers were placed on the left and right ankles, knees, and hips using adhesive tape and the receivers were further secured using Velcro bands to prevent slippage in the course of the trial. The data were sampled at 40 Hz for each 20-s trial, yielding a total of 800 data points per trial about the x (mediolateral), y (anteroposterior), and z (vertical suspensory) directions of the left and right ankles, knees, and hips. Thus for each trial, 18 columns of time series data were collected (x, y, z directions about the six effectors). For each trial, these time series were rescaled to the interval $[-1, 1]$ by subtracting the time-series mean and dividing by time-series maximum value. The data were subsequently stored on the PC for further analysis.

Two specific kinds of analyses were performed: (a) K-L transformation to determine the number of orthogonal dimensions that it takes to describe the data and (b) a Fourier spectral analysis to determine the frequency in the preferred mode of oscillations. With respect to (a), the effectors or joints in hula hooping are represented by an N-dimensional vector $Y = [(x_1, t), (x_2, t), (x_3, t) \ldots (x_n, t)]$, where x_i is the time series of an individual joint motion. Figure 1 shows 18 measured signals about the x, y, and z planes of the left and right ankles, knees, and hips, respectively, obtained from a single performer in experiment 1. Implementing the K-

L transformation entails computing C_{ij}, a matrix of M orthogonal vectors ($m \times m$ mutual information matrix) defined by

$$C_{ij} = \langle (x_i)(x_j) \rangle \tag{1}$$
$$= \frac{1}{N - m + 1} \sum_{n=1}^{N-m+1} (x_{N-m+i})(x_{N-m+j}).$$

For details on the derivation of the mutual information function, see Abarbanel (1996). The classic K-L expansion is a linear technique that uses only a covariance matrix for decomposing the recorded signals. The mutual information function used here is more accurate because of its sensitivity to nonlinearities in the data, robustness with respect to nonstationarity, lack of knowledge of data distributions, etc. (Eubank and Farmer 1996). The number of nonzero eigenvalues of the matrix C_{ij} represents the number of orthogonal components that can express the recorded 18-dimensional vector.

The optimality of the K-L method reduces the amount of information about a signal down to a reasonable number of independent eigenvalues that represent important characteristic features of the signal (Broomhead and King 1986; Uhl et al. 1995; Haken 1996). The first principal eigenvalue is taken to be along the direction with the maximum variance. The second principal eigenvalue is constrained to lie in the subspace perpendicular to the first. Within that subspace, it points in the direction of the maximum variance. Then, the third principal eigenvalue (if any exists) is taken in the maximum variance direction in the subspace perpendicular to the first two, and so on.

2.1.3 Procedure.
The participants were asked to balance the hula hoop at a pace most comfortable to them for 20 s. The data collection on a 20-s trial began as soon as the participant expressed that the oscillations were comfortable in terms of speed, smoothness, and stability. A total of ten trials of data were collected for each participant. (This procedure and the similar procedure of experiments 2 and 3 were approved by the Institutional Review Board of the University of Connecticut.)

2.2 Results

2.2.1 K-L modes
The K-L transformation was applied to the 18 rescaled time series of each trial of each participant. For each trial, we obtained the principal K-L eigenvalues, their corresponding eigenvectors, and the amount of variance in the original data that each of the eigenvectors explained. Table 1 shows, for each participant, the percentage of variance in the spatiotemporal pattern explained by each of the eigenvalues averaged over all trials. Inspection of Table 1 reveals that for all participants, in each case, the first five eigenvalues accounted for about 98.5% of the total variance in the kinematic patterns. The participants' means are shown in Fig. 2.

Table 1. Eigenvalues (λ) of the KL transformation matrix and the percentage of the total variance accounted for by each

Participant	λ_1	λ_2	λ_3	λ_4	λ_5
1	72.995	18.620	7.810	0.907	0.010
2	70.320	19.750	8.010	1.020	0.005
3	83.210	15.220	8.030	0.050	0.010
4	79.950	14.210	5.850	1.820	0.006
5	76.220	15.170	6.120	1.850	0.012
6	74.230	22.140	3.850	0.041	0.003
7	80.170	13.330	4.210	1.030	0.045

2.2.2 Spectral analysis. For each trial of each participant, the 18 rescaled time series and the first five eigenvectors obtained from the K-L transform were subjected to a fast Fourier transform (FFT).

The FFT revealed that, in general, the ankle, knee, and hip oscillated at approximately the same frequency while balancing the hoop under self-selected temporal conditions. The FFT of the eigenvectors of the K-L transform revealed that the dominant modes exhibited peaks at the same frequency as the mean frequency of the joints. Though participants showed variation in their preferred joint frequency, peaks were typically observed in the vicinity of 2 Hz (grand mean of 1.83 Hz), i.e., two full cycles every second, approximately 40 cycles per trial.

2.2.3 Projections. The K-L transformed data were projected back into the workspace of the original data (for a review of this procedure, see Daffertshofer et al. 2003). The lower panel of Fig. 2 shows the two leading projections ξ_1 and ξ_2 plotted against each other. It is important to note that the two projections when plotted together do not form a circle, suggesting that the decomposition did not just pick up two components that have a phase difference of $\pi/2$ that reflect sine and cosine functions. In fact, it is worthwhile to note that the two projections are actually operating in the same phase. This was verified by a relative phase analysis carried out between the two leading projections, ξ_1 and ξ_2.

The rescaled time series of projection was subjected to a Hilbert transform (Rosenblum and Kurths 1998) given by

$$H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} \, d\tau \;, \tag{2}$$

where $x(t)$ is a given time series. The relative phase measure $\phi(t)$ given in radians for two time series $x(t)$ and $y(t)$ is obtained by

$$\phi(t) = \arctan[(H_y it(t)x(t) - H_x(t)y(t))/(x(t)y(t)$$
$$+ H_x(t)H_y(t))] \;, \tag{3}$$

where $H_x(t)$ and $H_y(t)$ are the corresponding Hilbert transforms for $x(t)$ and $y(t)$, respectively. The Hilbert method assumes no specifics about the nature of the oscillators. A previous application was to cascade juggling (Post et al. 2000).

The relative phase analysis revealed that the leading projections (which in sum capture a very large
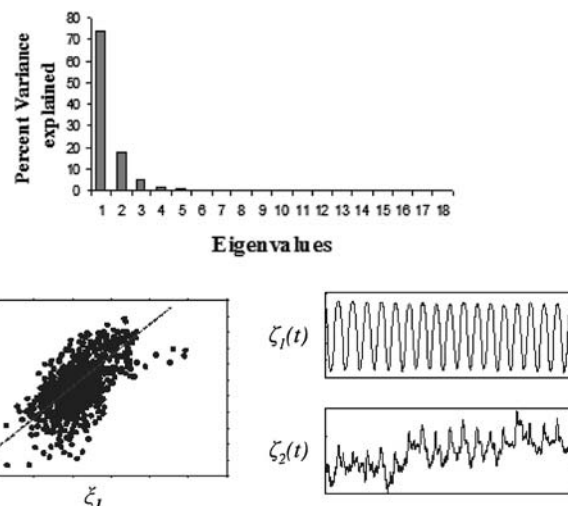
Fig. 2. Mean percent variance of the spatiotemporal pattern explained by each eigenvalue in Experiment 1 averaged over all participants and trials. The *bottom left panel* shows the projection of the first two modes plotted against each other, and the *bottom right* shows them separately

proportion of the data) were very closely linked in phase; across all projections, the mean relative phase was 0.11 ± 0.03 rad. The bottom panel of Fig. 2 illustrates a sample from one subject.

2.3 Discussion

The fact that 98.5% of the data are typically accounted for by the first five eigenvalues, and mostly by the first two, suggests that a low-dimensional description of the spatiotemporal pattern is viable. This low dimensionality is probably due to the high level of redundancy and similarity that exists in the data structure, as suggested by the FFT, which showed peaks at very similar values for ankle, knee, and hip. The organization of such movements might therefore involve a very simple control system of relatively few degrees of freedom. The results add to the existing findings of Haken (1996) and Post et al. (2000) that the K-L transform is a potentially useful strategy for understanding dimensionality reduction in biological movement systems.

The fact that a few orthogonal vectors can adequately describe a large data set of joint motions reveals some element of redundancy between elements in the cross-covariance matrix. The individual signals obtained from the effectors at which the data were recorded were highly

similar in structure and time evolution. If one were to start with the assumption that the oscillations about hip, knee, and ankle were three-dimensional sinusoids all at the same phase, amplitude, and frequency, one would end up with a perfectly symmetrical cross-covariance matrix and with one K-L mode. Specifically, a sinusoid of equivalent amplitude and frequency would be sufficient to describe the entire spatiotemporal pattern. On the contrary, if the oscillations were completely uncorrelated noisy signals, then one would expect to see very little redundancy in the cross-covariance matrix and there would be no possibility of achieving a low-dimensional reduction. In the extreme case one would require as many vectors as one started with to describe the spatiotemporal pattern and its organization. The projections show that the two leading K-L modes are rather closely related to each other in phase and frequency from inspection of their relative distributions (see bottom Fig. 2). The physics of the hoop suggests that two processes are required to sustain oscillations: a fore-aft oscillation component to sustain the hoop motions and a vertical regulatory component that provides the right impulse to keep a uniform force field over the plane of the hoop. Considering the biomechanical properties of the lower limb, one might hypothesize the following. The projections indicate that the first two K-L modes that account for most of the variance in the spatiotemporal patterns might have functional relevance in that the most dominant motion in hula hooping is in the fore-aft direction and the second most dominant one is that of the knee in the vertical direction.

3 Experiment 2: Mode modulation by variation in hoop size

In experiment 2, the focus is on the adaptation of the K-L modes identified in experiment 1 to different hoop sizes. Balancing a small hoop requires faster oscillations of smaller amplitude than are required in the balancing of a large hoop. How might these changes be reflected in the K-L modes? Frequency and amplitude are candidate control variables. As is well known from investigations of interlimb rhythmic coordination within the context of the Haken-Kelso-Bunz equation, manipulations of coupled frequency affect the mean value and variability of the relevant order parameter, relative phase (e.g., Turvey 1998). At increasingly higher frequencies, an interlimb coordination pattern may be sufficiently destabilized to give way to another pattern of higher stability (Kelso 1995). A possible consequence, therefore, of variation in hoop size is a qualitative change in the spectrum of K-L modes.

As noted, the low dimensionality detected in experiment 1 was implicated in the symmetry of the cross-covariance matrix. That symmetry reflected the symmetries among the component oscillations comprising the spatiotemporal activity of the lower limbs. An obvious expected consequence of variation in hoop size is variation in the uniformity (in-phase and frequency) of the elements that went into the mixing cross-covariance matrix. A change in relative contributions to the matrix's symmetry would arise from a change in the phasing of ankle, hip, and knee oscillations. Consequently, at issue in Experiment 2 were two questions: (a) Does the modal description of the spatiotemporal patterning of lower limb activity in hula hooping undergo systematic change with hoop size? (b) Are there corresponding systematic changes in the relations among the component oscillators (joint-joint phase relations) with change in hoop size?

3.1 Method

3.1.1 Participants. The four men and three women who participated in Experiment 1 returned to participate in Experiment 2.

3.1.2 Apparatus, data collection, and analyses. The apparatus and data acquisition/analysis procedures were largely the same as in Experiment 1. Three hula hoops (made by Wham-O Corporation, Torrance, CA) of diameters 80.4 cm (large), 75 cm (medium), and 70.4 cm (small) and weighing, respectively, approximately 200, 192, and 184 g were used.

Three specific kinds of analyses were performed: (a) K-L transformation to determine the number of orthogonal dimensions it takes to describe the data and the variance of the spatiotemporal pattern they explain, (b) Fourier spectral analysis to determine the preferred frequency across different diameters, and (c) Hilbert relative phase analysis of the relationship between the ankle, knee, and hip in the anteroposterior direction. The rescaled time series of each segment was subjected to a Hilbert transform as per the method described in the previous section (2.2.3).

3.1.3 Procedure. The procedure was basically the same as that of Experiment 1. There were 10 trials, each of 20 s, for each of the three diameters for a total of 30 trials per participant. Diameter was randomized across the 30 trials. On each trial, data collection began as soon as the participant indicated that the oscillations were comfortable in terms of speed, smoothness, and stability. The data were subsequently stored on the PC for further analysis.

3.2 Results

3.2.1 Mode frequency and amplitude. The 18 rescaled time series vectors were subjected to the K-L transformation. For each trial, FFT was conducted on the temporal evolution of the eigenvalue corresponding to the first eigenvector. The FFT revealed a difference in the mean peak frequencies for the three diameter conditions: large (1.84 Hz), medium (2.01 Hz), and small (2.19 Hz), $F(2, 12) = 10.45$, $p < 0.01$. Peak-to-peak amplitude on the dominant eigenvalue (mode) showed an effect of hoop size, $F(2, 12) = 11.37$, $p < 0.05$; large, 26.1 units; medium, 23.4 units; and small = 21.8 units.
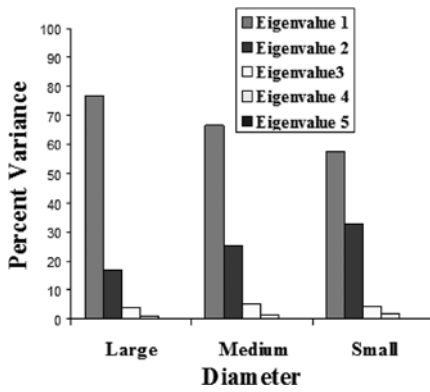
Fig. 3. Each eigenvalue plotted against percentage of variance explained for the large, medium, and small hoop-diameter conditions

The values are represented in abstract distance units because, due to rescaling, the modes cannot be represented in the units the data were collected in. Thus the manipulation of hoop size affected both mode frequency and mode amplitude.

3.2.2 K-L modes. For each trial of each participant, the principal K-L modes, their corresponding eigenvectors, and the amount of variance explained by each eigenvector were obtained. The amount of variance explained by the first five K-L modes (accounting for 98.5% of the total variance) was averaged across subjects for each hoop diameter condition. A 3 (diameters: large, medium, and small) $\times$ 5 (first five eigenvalues) ANOVA performed on the explained variance yielded a significant interaction, $F(8, 90) = 274.92$, $p < 0.0001$. Figure 3 shows the nature of this interaction. With decreasing diameter, the proportion of variance explained by the first mode decreased, whereas that explained by the second mode increased.

3.2.3 Projections. The modes projected onto the data ξ_1 and ξ_2 were visually very similar to the results reported in Experiment 1. In general, the two modes were closely linked in phase (grand mean = 0.15 ± 0.06 rad). However, the Hilbert relative phase between the two modes showed no statistical differences across the different hoop diameter conditions ($p > 0.05$).

3.2.4 Mean relative phase of participating joints. The Hilbert transform was used to compute relative phase between contralateral homologous joints and between ipsilateral joints for motions in the y direction. A 3 (joint-joint) $\times$ 3 (diameter) ANOVA performed on the mean relative phases between contralateral homologous joints (e.g., hip-hip, knee-knee, ankle-ankle) yielded no interaction and no main effects ($Fs < 1$). Separate analysis on ipsilateral phase relations of both the right and left lower limbs yielded significant interactions: $F(4, 24) = 77.99$, $p < 0.0001$ for the right limb and $F(4, 24) = 83.65$, $p < 0.0001$ for the left limb. The interactions are summarized in Fig. 4. The relative phases of knee-hip and knee-ankle were affected by
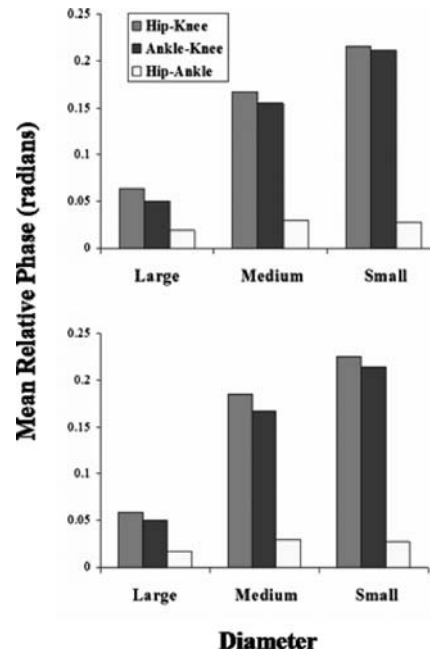

Fig. 4. Mean relative phase (in radians) between the joints as a function of hoop size in Experiment 2. The *upper panel* is the right limb, the *lower panel* is the left limb

hoop size to a greater degree than the relative phase of hip-ankle. As hoop size was reduced, knee-hip and knee-ankle deviated from a relative phase of 0 rad, whereas hip-ankle remained unchanged at a relative phase of approximately 0 rad. Following the Haken-Kelso-Bunz model (Haken et al. 1985), the joint-joint mean relative phase defines the stability or fixed point or equilibrium of joint-joint coupling. In-phase (0 rad) is the most stable phase relation.

3.2.5 Standard deviation of relative phase of participating joints. ANOVA found no interaction or main effects for the contralateral homologous joints ($Fs < 1$). The results for ipsilateral joint-joint coordinations are shown in Fig. 5. Inspection suggests that variability for both right and left limbs was inversely related to hoop size with the change in variability due to hoop size greater for knee-hip and knee-ankle than for hip-ankle. ANOVA confirmed these interactions: $F(2, 12) = 3.21$, $p < 0.05$ for the right limb and $F(2, 12) = 4.52$, $p < 0.05$ for the left limb.

3.3 Discussion

Given that 95% and above of the variance was accommodated at each hoop size by the first two K-L modes, it is reasonable to conclude, in agreement with Experiment 1, that two DF suffice to capture the spatiotemporal pattern of the lower limbs. Whatever the nature of these two modes or DF, it is evident that their relative contributions to hula hooping depended on hoop size, specifically, the demands that hoop size imposed upon movement frequency and movement
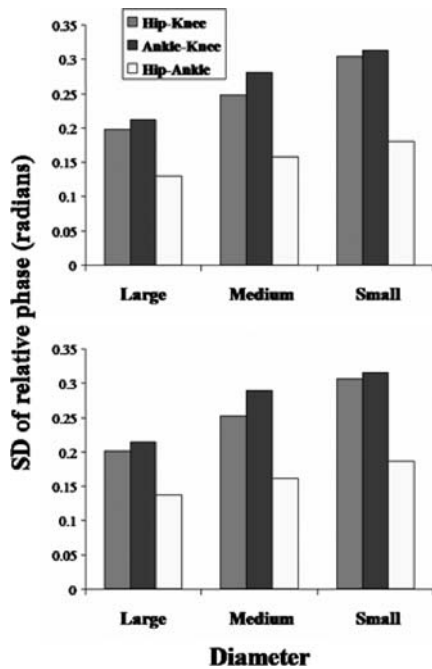
Fig. 5. Standard deviation of relative phase as a function of hoop size in Experiment 2. The *upper panel* is the right limb, the *lower panel* is the left limb

amplitude. Whereas the number of modes needed to explain the spatiotemporal pattern did not change, the relative amount of variance explained by the two modes did change. The large diameter condition associated with the largest amplitude and lowest frequency proved to be most dominated by the first mode and least dominated by the second mode. With respect to the mathematics of the K-L transform, some aspect of the symmetry of the cross-covariance matrix depended on hoop diameter.

The large diameter condition also proved to be the most stable as reported spontaneously by participants and as substantiated by Hilbert relative phase and its variability. The mean relative phase of the ipsilateral hip-knee coordination was approximately 3° with the large hoop and approximately 9° and 12° with the intermediate and small hoop, respectively. Ipsilateral joint coordinations were closest to in-phase for the large hoop. They were also least in variability. The standard deviation for the large hoop was approximately 11° compared to 14° and 17° for the intermediate and small hoop, respectively.

Research on interlimb coordination has shown that a relative phase of 0° is more stable, as measured by standard deviation and relaxation time, than nonzero relative phase, with the degree of instability amplifying with increasing deviation from 0° (e.g., Sternad et al. 1996). Further, that research has shown that deviations from 0° are induced by an asymmetry in the contributions of the component oscillators to the coordination (that is, coupled oscillator) dynamics (see reviews by Kelso 1994; Schmidt and Turvey 1995). Given such an asymmetry, its destabilizing effect on the coordination dynamics is further magnified by an increase in

movement frequency (Amazeen et al. 1998). On the basis of these findings, one might make two inferences about the present data. First, that a small inequality exists in the contributions of knee and ankle oscillators to knee-ankle coordination and in the contributions of knee and hip oscillators to knee-hip coordination. Second, that these inequalities are least in the large hoop condition and become magnified with the reduction in hoop size and the concomitant increase in movement frequency and/or amplitude. A reasonable guess is that the contribution of the knees to the coordination dynamics of the lower limbs in hula hooping was exaggerated, relative to the contribution of the ankles and hips, by the increase in movement frequency and/or amplitude induced by the stability demands of the smaller hoops. It is significant, in this regard, that hip-ankle relative phase and its variability was largely indifferent to hoop size. The knee, apparently, assumes a key role in the organization of the dynamics. That hip-knee relative phase and ankle-knee relative phase (mean and SD) changed most points to a significant vertical modulation of CoM with decreasing hoop size.

A possible interpretation of the two dominant modes follows from a consideration of the relative phase data in Figs. 4 and 5 with the mode data of Fig. 3. Namely, it seems that the first K-L mode projection is associated with a dynamical subsystem encompassing hip and ankle, and the second K-L mode projection is associated with a separate dynamical subsystem encompassing hip, knee, and ankle. On this speculation, as hoop size decreased, the hip-knee-ankle dynamics rose in prominence and the hip-ankle dynamics declined in prominence. In view of the basic physics of hula hooping, the aforementioned two dynamical systems can be regarded as fulfilling the two duties of the carefully regulated impulse, namely, to change the horizontal and vertical components of the hoop's angular momentum. In close to ideal conditions, here approximated by the large hoop, the dynamical subsystem (encompassing hip, knee, and ankle) responsible for provision of the vertical component is of reduced significance relative to the subsystem (encompassing hip and ankle) responsible for the horizontal component. As conditions depart from the ideal (a decrease in hoop size in the present experiment), the dynamical subsystem responsible for controlling the change in the vertical component of the momentum assumes a larger role.

The relative phase analysis enhances understanding of hula hoop dynamics in one final sense. If in-phase defines the most stable intersegmental coordination, then the hip-ankle coordination can be viewed as persisting in this most stable state over variation in hoop size. In contrast, as noted above, one can view the two coordinations involving the knee as being detuned by hoop-size variation. The change in the symmetry or uniformity of the cross-covariance matrix as revealed by the K-L transformation may be seen, therefore, as the consequence of detuning, manifest as a shift in the stability points of the hip-knee and ankle-knee coupled oscillations.
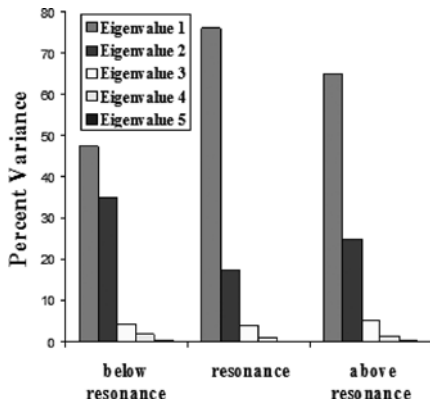
Fig. 6. Each eigenvalue plotted against percentage of variance explained for the three frequency conditions ($0.9\omega_0$, ω_0, $1.1\omega_0$) in Experiment 3
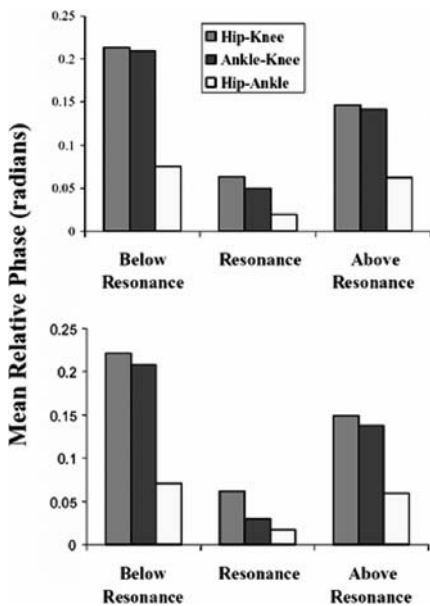

Fig. 7. Mean relative phase (in radians) between the joints as a function of frequency in Experiment 3. The *upper panel* is the right limb, the *lower panel* is the left limb

4 Experiment 3: Modal organization at preferred frequency

An intimate connection has been observed between movement timing and stability (for summaries see Kelso 1995; Amazeen et al. 1998). When timing demands are made on a system performing at speeds above and below resonance, loss in stability is observed and is typically attributed to emergent interactions at lower levels of the limb's neural and physical dynamics (Hatsopoulos 1996; Hatsopoulos and Warren 1996). There is evidence that the freely elected tempo of rhythmic limb movements is that of resonance (Turvey et al. 1988). Benefits of performing at resonance include optimal metabolic cost (Holt et al. 1995) and cycle-to-cycle stability and reproducibility (Rosenblum and Turvey 1988). Analysis of resonant and nonresonant rhythmic movements

through phase-space reconstruction methods (Abarbanel 1996) reveals additional benefits of resonance, namely, minimal active degrees of freedom, minimum noise, and maximal predictability (Goodman et al. 2000).

The results of Experiment 2 indicated that the low-dimensional modal description of the spatiotemporal pattern in hula hooping underwent a systematic change as a function of hoop size. The objective of Experiment 3 was to see if the K-L modes and the phasing relations between the component oscillators (hips, knees, ankles) were affected by frequency manipulations scaled to resonance. For each of the seven participants, the mean self-selected frequency of the large hoop in Experiment 2 was designated as the resonant frequency. Then, for each participant a frequency above and a frequency below resonance was defined as fractional multiples of the preferred (hence operationalized as resonant) frequency. Hula hooping at all three frequencies (each signaled by a metronome) was conducted with the large hoop of Experiment 2. The primary expectation was that the spectrum of K-L modes would be most simple at resonance, in the sense that the first mode would accommodate a disproportionate amount of the spatiotemporal variance. With deviations from resonance, the contribution of the second K-L mode would become more pronounced. Behind this expectation were the assumptions that (a) a performer's preferred frequency (resonance) is that at which the required change in the vertical component of the angular momentum is minimal and (b) the first and second K-L modes are synchronous or very nearly so. Given (a) and (b), one would expect that the hypothesized subsystems encompassing hip-ankle and hip-knee-ankle would deviate significantly from a relative phase of 0 rad in the nonresonant conditions.

4.1 Method

4.1.1 Participants. The seven participants of Experiments 1 and 2 were used again in Experiment 3.

4.1.2 Apparatus, data collection/reduction, procedure. These replicated in most respects those of the preceding experiments. The large hoop used in Experiment 2 (diameter 80.4 cm, 200 g) was used in Experiment 3. Participants were instructed to follow three frequencies as dictated by a metronome on a portable Macintosh computer. Each participant's preferred self-selected frequency (the mean frequency of the large hoop trials of Experiment 2) was chosen as the resonance frequency ω_0. For the seven participants, the values of ω_0 (in Hz) were: 1.73, 1.89, 2.01, 1.65, 2.03, 1.62, and 1.85. For each participant, the metronome was set to present frequencies of $0.9 \, \text{g}\omega_0$ (below resonance), ω_0 (at resonance), and $1.1\omega_0$ (above resonance) in a randomized order. Ten trials each for the three frequencies were presented in random order, yielding 30 trials per participant.

4.2 Results

4.2.1 K-L modes. The 18 rescaled time series vectors were subjected to the K-L transformation. For each trial, the principal K-L eigenvalues, their corresponding eigenvectors, and the amount of variance of each of the eigenvectors explained of the original data were obtained. The amount of variance explained by the first five eigenvalues (which accounted for 98.5% of the total variance) was averaged across participants for each hoop diameter condition. A 3 (frequencies: below resonance, at resonance, and above resonance) × 5 (first five eigenvalues) ANOVA was performed on the variance explained. A significant interaction was found between the factors of frequency and eigenvalues, $F(8, 90) = 54.64$, $p < 0.0001$. Figure 6 shows the nature of this interaction.

The data pattern shows qualitatively similar changes across the frequency conditions as observed in the manipulation of hoop size in Experiment 2. An examination of Fig. 6 shows that the proportion of variance explained by the first K-L mode is greater at resonance than away from resonance. Conversely, the proportion of variance accommodated by the second K-L mode was less at resonance than away from resonance.

4.2.2 Projections. The modes projected onto the data yielded results similar to those of Experiments 1 and 2. However, this time there was a clear statistical difference between the relative phase between ξ_1 and ξ_2. The mean relative phase between the modes at resonance was highest at below resonance (mean of 0.22 rad), followed by the above resonance condition (mean of 0.17 rad) and the resonance condition (mean of 0.15 rad).

4.2.3 Relative phase analysis of the joints. Hilbert relative phase transforms were performed in a manner similar to that described in experiment 2. A 3 (joint–joint) × 3 (frequency) ANOVA performed on the mean relative phase between the contralateral homologous joints (e.g., hip–hip, knee–knee, ankle–ankle) yielded no significant effects ($Fs < 1$). Three relative phase quantities were obtained for each limb (hip–ankle, hip–knee, and ankle–knee). A 3 (joint-joint relations) × 3 (frequency) ANOVA was performed on the mean and standard deviation of the relative phase separately. For mean relative phase, a significant interaction was found between frequency and joint relations, $F(4, 24) = 21.98$, $p < 0.0001$ for the right limb and $F(4, 24) = 32.98$, $p < 0.0001$ for the left limb. This interaction is summarized in Fig. 7, with the upper panel showing the effects for the right limb and the lower panel showing the effects for the left limb. Inspection of Fig. 7 shows that the differences in relative phase for hip-ankle and the two joint–joint relations involving the knee were magnified by the nonresonant conditions, more so for below resonance than for above resonance. Both joint relations involving the knee exhibited significant shifts from in-phase at the two nonresonance frequencies. The ANOVA on the standard deviation of relative phase similarly revealed a strong interaction between
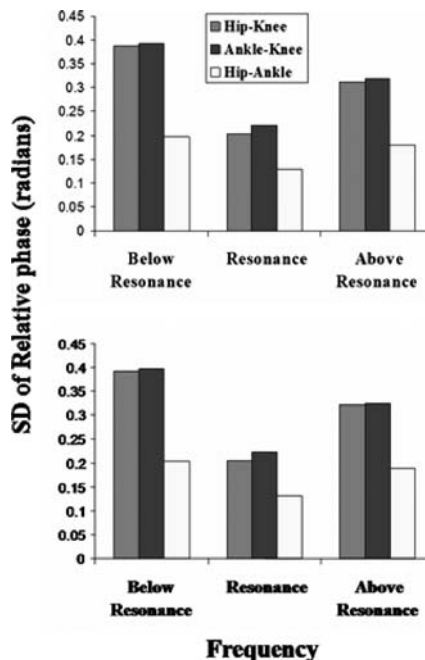

Fig. 8. Standard deviation of relative phase between the joints as a function of frequency in Experiment 3. The *upper panel* is the right limb, the *lower panel* is the left limb

frequency and joint relations, $F(4, 24) = 16.39$, $p < 0.001$ for the left limb and $F(4, 24) = 22.67$, $p < 0.001$. The details of this interaction are shown in Fig. 8. Thus, the resonance condition showed the least mean and standard deviation for relative phase (closest to in-phase) and the below resonance condition showed the highest (farthest from in-phase).

4.3 Discussion

Consonant with the results of Goodman et al. (2000) for rhythmic movements of a single joint, the results of the present experiment suggest that the rhythmic movements of multiple joints are least complex and most stable at resonance. Here resonance refers not to an intrinsic feature of the joints but to the global dynamics of hula hooping. It is the tempo at which the body's oscillations are most attuned to the horizontal oscillations of a hoop of particular diameter. Quite possibly, the resonance of hula hooping is expressible in terms of the phase relation between the two primary K-L modes. It is, perhaps, the frequency at which the two modes are most closely synchronized.

It was suggested on the basis of Experiment 2 that the two modes refer independently to the dynamics of imparting a change in (a) the horizontal component of the hoop's angular momentum and (b) the vertical component of the hoop's angular momentum. Intuitively, the observation that the second K-L mode was most prominent below resonance seems to be in keeping with the idea that the second K-L mode provides the vertical component. At a slower nonpreferred pace, it

should be difficult to maintain a larger hoop in motion in a horizontal plane. Success would require a more complicated patterning of the vertical component of the impulse forces to counter the hoop's strong tendency to fall.

The flexibility of the relation between the lower limbs' two K-L modes evident in both Experiments 2 and 3 is probably reflective of the constraint of pattern stability imposed from the level of the hoop-body coupling. Variation in circumstances – hoop size, imposed frequency – is associated with variation in the hoop-body relation that is temporally stable. Patently, the same hoop-body relation cannot satisfy all circumstances under which hula hooping is performed. A relation that is temporally stable under one set of circumstances will be temporally unstable under others. In sum, a reasonable interpretation of the changes seen in the contributions of the two primary K-L modes is that they constitute necessary adjustments to achieve circumstance-dependent pattern stability.

5 General discussion

A primary challenge in the investigation of movement coordination is discovering how high-dimensional state spaces are compressed into low-dimensional control spaces. This problem has been termed "Bernstein's Problem" (Kugler et al. 1980; Turvey 1990) in recognition of Bernstein's (1967, 1996) pioneering studies directed at coordination of movement as "[the] process of mastering redundant degrees of freedom of the moving organ, in other words its conversion to a controllable system (Bernstein 1967, p. 127)." In the present research, we investigated the "organization of the control of the motor apparatus (Bernstein 1967, p. 127)" in hula hooping. The results allow the tentative conclusion that, in performing this particular task, the multiple DF of the state space of the lower limbs were compressed to two control DF.

A brief reprise of Experiments 1–3 clarifies the basis for the preceding conclusion. The oscillations about the hip, knee, and ankle joints in performing the task are highly redundant. The spatiotemporal pattern they exhibit is a consequence of constraints due to biomechanical linkages and functional relations among the component oscillations as dictated by the common goal of producing sustained motions of the hoop in a nearly horizontal plane. Application of the K-L transform to the data of Experiment 1 revealed that two modes were sufficient to accommodate most of the variability in the spacing and timing of the segmental motions. This low-dimensional description was made possible, in part, by the symmetry of the cross-covariance or mutual information matrix and the redundancy seen in the data. This description underwent qualitative changes with manipulations of hoop diameter and resonance-scaled frequency manipulations in Experiments 2 and 3, respectively. Though the number of orthogonal vectors required to describe the system did not change, the proportion of variance in the spatiotemporal pattern accounted for by the first eigenvalue was lower with a decrease in hoop size and when hula hooping was conducted away from resonance. Importantly, the converse was true for the second eigenvalue.

Hoop diameter and oscillation frequency relative to resonance may be considered as important control variables for the dynamics of hula hooping. A superficial similarity between these control variables and control parameters (from the synergetics approach) might exist. Manipulation of such variables provides an opportunity to study a range of effects on the successful performance of the task at hand, and not on theoretically interpreting what such variables might mean in the broader synergetic context. In Experiments 2 and 3, small changes in movement detail were expressed as qualitative changes in the movement organization as a whole, in a manner suggested by Bernstein (1967, p. 23). Put differently, the dimensionality of the control space underwent graceful change as a function of changes in variables nonspecific to the system itself.

It is generally the case that few modes are sufficient to account for the variation in most spatially distributed coherent systems (Kelso 1995; Haken 1996; Post et al. 2000). Some of the referenced studies have tried to make contact with the nature of the physiological apparatus on which those modal descriptions are realized. For example, Haken (1996) concluded that the learning of the pedalo mostly involved upper body movements. This was achieved on the basis of projecting the eigenvectors onto the measured spatiotemporal pattern. Post et al. (2000, 2003), using similar projection techniques, found that the first two eigenvalues were almost always associated with the vertical direction. This finding suggested that the control structure for three-ball cascade juggling might largely be concerned with the gravitational field, consistent with the findings of van Santvoord and Beek (1996). While projection techniques are largely exploratory, to confirm what the principal components or modes correspond with in any functional sense, one can experimentally manipulate variables that show qualitative reorganization in the modes and perform subsequent complementary analysis on the data.

The preceding investigations inspire the question: What substantive bearing might the two dominant K-L modes of the kinematics of the lower limbs have on the control structure of hula hooping?

A step toward addressing this question was taken in the present research by examining the relative phase between the component oscillations at the joints. Relative phase was elected to be a candidate variable behind the change in the cross-covariance matrix that could have yielded qualitative changes in the K-L modes over experimental manipulations. The relative phase analysis showed that, while the hip-ankle relation stayed close to 0 rad or in-phase, the joint–joint relations involving the knee drifted away from in-phase. In the discussions above, it was suggested that this effect could be due to a detuning or "imperfection" in the relations between the component oscillators about the joints. In particular, the phase relations involving the knee showed marked shifts

in equilibria with changes in the nonspecific control parameters.

Following careful analyses of the relative variation in the modes, we performed supplementary relative phase analyses on the joints that contribute to the mixing matrix that went into the K-L transformation. We have reason to believe that the two primary K-L modes identified in the present research might indicate the presence of two functional synergies or coordinative structures that encompass the lower limbs while the body is engaged in hula hooping. While fore-aft oscillations are very important in providing the requisite impulse for imparting angular momentum to the hoop, the balance of the hoop in a horizontal plane against the direction of gravity is very likely to involve vertical suspensory activity, which is commonly understood to involve rotations about the knee joint. An argument can be made, therefore, that the two dominant modes may indicate two functional regimes or subsystems. Said differently, the changing geometric relationships in the limbs serve two functions: one to sustain fore-aft oscillations (largely involving hip-ankle movements) and another to organize the vertical suspension of the body (largely involving knee joint rotations). When performance was challenged under manipulations such as hoop size and frequency, the aforementioned organizations underwent systematic change. While the hip-ankle subsystem changed little, the vertical suspensory organization altered its geometric relations with respect to the former, largely in order to meet the new demands of balancing the hoop.

Evidence for the independence of lower limb topological organizations exists. Winter et al. (1996) found that in quiet stance fore-aft oscillations or anteroposterior (AP) sway was largely due to plantarflexion and dorsiflexion about the ankle joint and mediolateral (ML) sway was generated by abduction and adduction about the hip joints. Balasubramaniam and Turvey (2000) and Balasubramaniam et al. (2000) reported that AP and ML sway (i.e., hip and ankle rotations) may be independently modulated to facilitate the performance of posture-related precision tasks such as archery and that the synergy for upright posture involves the controlled assembly of two distinct systems with reciprocally related dynamics. The two orthogonal modes obtained by K-L decomposition might therefore indicate some aspect of two independent synergies whose superposition captures the entire spatiotemporal pattern of the lower limbs in hula hooping.

A more comprehensive understanding of the biological system as a "hula hooper" might require a mode decomposition analysis that investigates the oscillations of the body and the hula hoop considered as a system together.

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Appendix A: Order parameter identification

For an effective order parameter identification and analysis of any system, one has to contend with the notion of spatiotemporal states. A system is characterized by several quantities x_i, $i = 1, 2, \ldots, n$, forming a state vector Y. Since we consider a spatially distributed system, the state vector depends on the spatial coordinates x as well as time t. Accordingly, the spatiotemporal process of the system is characterized by n values of the state vector $y(x, t)$ at each point x in space and time t as represented by (4):

$$Y(x, t) = [Y_1(x, t) \ldots \ldots, Y_n(x, t)] \ . \tag{4}$$

The temporal evolution of a dynamical system can be described on the basis of time evolution equations. An evolution equation relates the temporal derivative of the state vector to a nonlinear function of the state vector as well as its spatial derivatives given by a paradigmatic equation of the form

$$\frac{\partial}{\partial t} y(x, t) = L(y; \nabla(y); \dot{y}) \ , \tag{5}$$

where $\nabla(Y)$ is a nonlinear function of y and is its spatial derivative. Such an equation allows one to calculate the state vector $Y(x,t)$ at a time t from knowledge of an initial condition, namely, the state vector at a previous time t_0. In systems that engage in cooperative behavior, the subsystems at different locations x_i do not evolve independently. If the whole system behaves in a collective way, then the state vector takes the form

$$Y(x,t) = Y[x, u(t)] \; . \tag{6}$$

Instead of specifying the state vector at each point in space and time, the pattern forming process is characterized by a finite set of variables $u_i(t)$, $i = 1, 2, \dots n_u$ called order parameters, where $\underline{c}$ is a control parameter. A dynamical system for $u_i(t)$ takes the form

$$\dot{u}i(t) = f(ui(t); c) \; . \tag{7}$$

Thus, instead of investigating (5) we examine a system of reduced dimensionality as given in (7).